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PREFACE

The First South-East European Conference on Computational Mechanics was held at the University of Kragujevac, Serbia, June 28-30, 2006.

Following the worldwide growing success of International and National Conferences on Computational Mechanics, the National Associations of Computational Mechanics of the South-East European Countries decided to launch an initiative for the organization of South-East European Conferences on Computational Mechanics every three years.

The aim of the conferences is to promote achievements in Computational Mechanics in the South-East European Region, to encourage research and development among young researchers, to stimulate education in Computational Mechanics at universities and to disseminate modern trends among scientists and engineers in the growing field of Computational Mechanics.

This First Conference was coordinated by the Yugoslav (Serbian) Society of Mechanics in cooperation with the Greek Associations for Computational Mechanics and Associations of Mechanics of other South-East European countries. The Conference was held under the auspices of the European Community on Computational Methods in Applied Sciences (ECCOMAS) and the International Association for Computational Mechanics (IACM). The organizers of the Conference was the University of Kragujevac, Faculty of Mechanical Engineering and the Center for Scientific Research of the Serbian Academy of Science and Art, with the support of the Ministry of Science and Environmental Protection of Serbia and the city of Kragujevac.

The response of the authors from the South-East European Region, and from many countries all over the world (authors from 31 countries), was very positive. Over 90 lectures were presented and a large number of young researchers participated.

The Proceedings of full-length papers is published as a book. The Proceedings and Book of Abstracts are available on CD. We have divided the papers and abstracts into the following sections:

1. General Lectures
2. Computational Methods
3. Solid Mechanics
4. Field and Coupled Problems
5. Biomechanics
6. Applications

The Scientific Committee decided to restrict the size of papers to 10 pages.

We believe the presented papers in the Proceedings offer not only a variety of subjects, but also a solid review of modern approaches in solving currently attractive problems in Computational Mechanics, as well as in Applied Medical Sciences. Also, we are certain that the Conference will foster the development and application of modern computational methods and will contribute to the enhancement of engineering education in the Universities of the Region.

It has been a great pleasure preparing the Conference and on behalf of the Scientific Committee and Organizing Committee, we would like to thank the Ministry of Science and Environmental Protection of Serbia and the city of Kragujevac for their support. We are thankful to our colleagues and collaborators at the Center for Scientific Research of Serbian Academy of Science and Art and the University of Kragujevac, as well as the Faculty of Mechanical Engineering and our friends from the city of Kragujevac, the University Choir "Liceum" and the "Student Cultural Center", for their enormous efforts to make the Conference successful and pleasant.

5 June 2006, Kragujevac, Serbia — Milos Kojic and Manolis Papadrakakis

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SEECCM 06

First
South-East European
Conference on
Computational Mechanics

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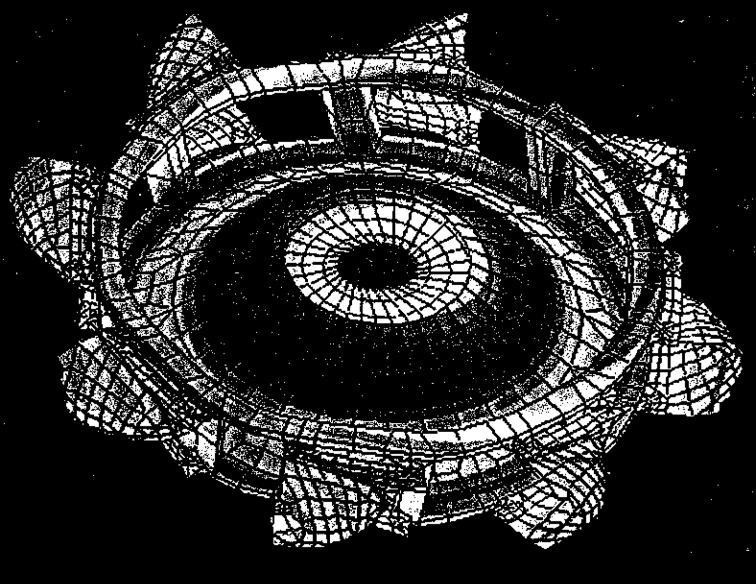

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The 3D Contact Problems Based on the Penalty Method

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Abstract

This paper presents model based on the penalty method for 3D contact problems with friction. The friction forces are assumed to follow the Coulomb's law, with a slip criterion treated in the context of a standard return mapping algorithm. The development includes exact linearization of the statement of virtual work, which enables optimal convergence properties for Newton-Raphson solution strategies, and which appears to be highly desirable for general robustness of implicit finite element techniques. Standard shape routines are used for the detection of contact between previously separate meshes and for the application of displacement constraints where contact has been identified.

The numerical model has been implemented into an version of the computational finite element program PAK. Numerical examples that illustrate the performance of the procedure are presented.

Key words: contact problem, penalty method, Coulomb's law

1. Introduction

Contact takes part in a wide range of engineering problems such as the interaction between soil and foundations in civil engineering, general bearing problems as well as bolt and screw joints, these are small deformation contact problems. On contrary, the impact of cars, car tire-road interaction and metal forming are large deformation contact problems. Here nonlinear material laws, damage, dynamic fatigue, friction, wear, etc. must be take into account to design optimal components and assemblies.

Out of the various real contact problems only very few can be evaluated analytically. Since the finite element method was developed at the beginning of the computer age, complex structural problems could have been solved.

The effective application of finite element contact solvers demands a high degree of experience since the general robustness and stability cannot be guaranteed. For this reason the development of more efficient, fast and stabile finite element contact discretizations is still a hot topic, especially due to the fact that engineering applications become more and more complex.

The aim of this paper is to provide framework or contact problems with friction, based on the penalty method. The penalty formulation has the advantage that it is purely geometrically based and therefore no additional degrees of freedom must be activated or inactivated. Numerical example are shown to present that algorithm can be successfully applied to contact problems.

2. Formulation of the multi-body frictional contact problem

General a contact can occur between: the deformable body and the rigid obstacle; between two deformable bodies or as a self-contact. In this paper contact between two deformable bodies is considered. As the configuration of two bodies coming into the contact is not a priori known, contact represents a nonlinear problem even when the continuum behaves as a linear elastic material.

2.1 Contact kinematics

Two bodies are considered: $B^{(1)}$ and $B^{(2)}$, Fig. 1. We will denote as the contact surface $\Gamma_C^{(i)}$ the part of the body $B^{(i)}$ such that all material points where contact may occur at any time t are included.

Using a standard notation in contact mechanics we will assign to each pair of contact surfaces involved in the problem as slave and master surfaces. In particular, let $\Gamma_C^{(1)}$ be the slave surface and $\Gamma_C^{(2)}$ be the master surface. Condition which should be satisfied is that any slave particle may not penetrate the master surface.

Let $\bar{\mathbf{x}}$ be the projection point of the current position of the slave node $\mathbf{x}^k$ onto current position of the master surface $\Gamma_C^{(2)}$, defined as

$$\frac{\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)}{\left\|\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)\right\|}\cdot\bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2)=0 \tag{1}$$

where $\alpha=1,2$ and $\bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2)$ are the tangent covariant base vectors at the point $\bar{\mathbf{x}}$. These tangents are defined using following relationships:

$$\bar{\mathbf{a}}_1 = \left.\frac{\partial \bar{\mathbf{x}}}{\partial \xi^1}\right|_{\xi^1=\bar{\xi}^1,\xi^2=\bar{\xi}^2}, \quad \bar{\mathbf{a}}_2 = \left.\frac{\partial \bar{\mathbf{x}}}{\partial \xi^2}\right|_{\xi^1=\bar{\xi}^1,\xi^2=\bar{\xi}^2} \tag{2}$$

The relation (2) can be written as:

$$\bar{\mathbf{a}}_\alpha = \bar{\mathbf{x}}_{,\alpha}\left(\bar{\xi}^1,\bar{\xi}^2\right) \tag{3}$$

The definition of the projection point allows us to define the distance between any slave node and the master surface. The normal gap or the penetration g_N for slave node k is defined as the distance between current positions of this node to the master surface $\Gamma_C^{(2)}$:

$$g_N = (\mathbf{x}^k - \bar{\mathbf{x}})\cdot\bar{\mathbf{n}} \tag{4}$$

where $\bar{\mathbf{n}}$ refers to the normal to the master face $\Gamma_C^{(2)}$ at point $\bar{\mathbf{x}}$ (Fig. 1). Normal to be defined using tangent vectors at the point $\bar{\mathbf{x}}$

$$\bar{\mathbf{n}} = \frac{\bar{\mathbf{a}}_1 \times \bar{\mathbf{a}}_2}{\|\bar{\mathbf{a}}_1 \times \bar{\mathbf{a}}_2\|} \tag{5}$$

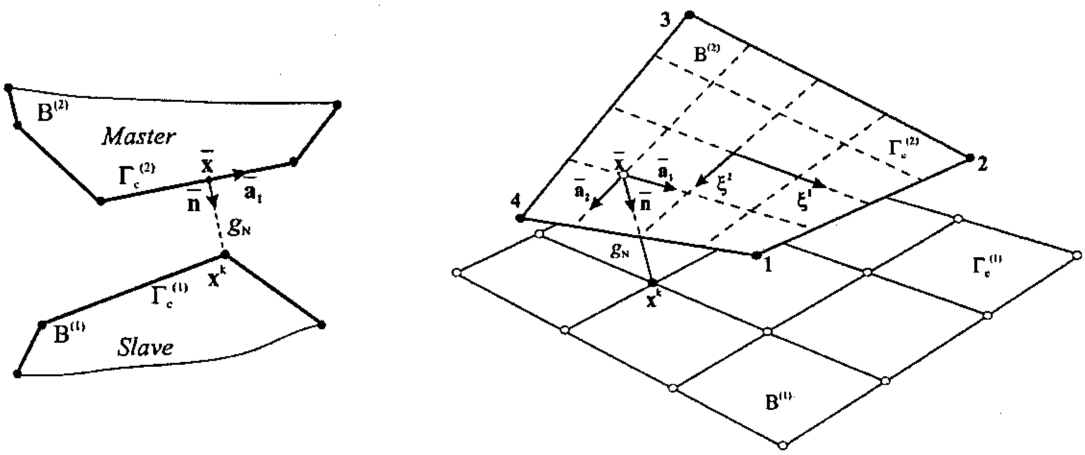

Fig. 1. Geometry of the 2D and the 3D node-to-segment contact element

This gap (4) gives the non-penetration conditions as follows

$$\begin{cases} g_N = 0 & \text{perfect contact} \\ g_N > 0 & \text{no contact} \\ g_N < 0 & \text{penetration} \end{cases} \tag{6}$$

If the analyzed problem is frictionless, function (6) completely defines the contact kinematics. However, if friction is modeled, tangential relative displacement must be introduced. In this case the sliding path of the node $\mathbf{x}^k$ over the contact surface $\Gamma_C^{(2)}$ is described by total tangential relative displacement as

$$g_T = \int_{t_0}^{t}\|\dot{\mathbf{g}}_T\|dt = \int_{t_0}^{t}\left\|\dot{\bar{\xi}}^\alpha\bar{\mathbf{a}}_\alpha\right\|dt = \int_{t_0}^{t}\sqrt{\dot{\bar{\xi}}^\alpha\dot{\bar{\xi}}^\beta a_{\alpha\beta}}\,dt \tag{7}$$

in time interval from t_0 to t.

The time derivatives of parameter $\bar{\xi}^\alpha$ in equation (7) can be computed from the relation (1), [8]. In the geometrically linear case we obtain the following result

$$\frac{d}{dt}\left[(\mathbf{x}^k - \bar{\mathbf{x}})\cdot\bar{\mathbf{a}}_\alpha\right] = \left[\dot{\mathbf{x}}^k - \dot{\bar{\mathbf{x}}} - \bar{\mathbf{a}}_\beta\dot{\bar{\xi}}^\beta\right]\bar{\mathbf{a}}_\alpha = 0 \tag{8}$$

which yields

$$\bar{a}_{\beta\alpha}\dot{\bar{\xi}}^\beta = \left[\dot{\mathbf{x}}^k - \dot{\bar{\mathbf{x}}}\right]\cdot\bar{\mathbf{a}}_\alpha = \dot{g}_{T\alpha} \tag{9}$$

where $\bar{a}_{\alpha\beta} = \bar{\mathbf{a}}_\alpha\cdot\bar{\mathbf{a}}_\beta$ is the metric tensor in point $\bar{\mathbf{x}}$ of the master surface $\Gamma_C^{(2)}$. From the equations (7) and (9) we can deduce the relative tangential velocity at the contact point

$$\dot{\mathbf{g}}_T = \dot{\bar{\xi}}^\alpha\bar{\mathbf{a}}_\alpha = \dot{g}_{T\alpha}\bar{\mathbf{a}}^\alpha \tag{10}$$

2.2 Constitutive equations for contact interface

For mathematical and computational modeling the surface characteristics have to be put into a constitutive interface constraint.

A contact stress vector $\overline{\mathbf{t}}$ with respect to the current contact interface $\Gamma_C^{(2)}$ can be split into a normal and tangential part.

$$\overline{\mathbf{t}} = \overline{\mathbf{t}}_N + \overline{\mathbf{t}}_T = \overline{t}_N \overline{\mathbf{n}} + \overline{t}_{T\alpha} \overline{\mathbf{a}}^{\alpha} \tag{11}$$

where $\overline{\mathbf{a}}^{\alpha}$ is contravariant base vector. The stress acts on both surfaces obeying the action-reaction principle: $\overline{\mathbf{t}}(\overline{\xi}^1, \overline{\xi}^2) = -\mathbf{t}$ in the contact point $\overline{\mathbf{x}}$. The tangential stress $\overline{t}_{T\alpha}$ is the zero in the case of frictionless contact. For contact one has the condition $\overline{t}_N < 0$. If there is not penetration between the bodies, then relations $g_N > 0$ and $\overline{t}_N = 0$ hold. This leads to the statements

$$g_N \geq 0, \quad \overline{t}_N \leq 0, \quad \overline{t}_N g_N = 0 \tag{12}$$

which are known as Kuhn-Tucker conditions.

In tangential direction a distinction is made between stick and slip. As long as no sliding between to bodies occurs, the tangential relative velocity is zero. If the velocity is zero, also the tangential relative displacement (7) is zero. This state is called stick case with the following restriction:

$$\dot{\mathbf{g}}_T = \mathbf{0} \Leftrightarrow \mathbf{g}_T = \mathbf{0} \tag{13}$$

A relative movement between two bodies occurs if the static friction resistance is overcome and the loading is large enough such that the sliding process can be kept. Therefore the relative sliding velocity, respectively the sliding displacement, shows in opposite direction to the friction force. With this the tangential stress vector is restricted as follows:

$$t_{T\alpha}^{sl} = -\mu \left| \mathbf{t}_N \right| \frac{\dot{g}_{T\alpha}^{sl}}{\left\| \dot{\mathbf{g}}_T^{sl} \right\|} \tag{14}$$

where μ is friction coefficient. In the simplest form of Coulomb's law (15), μ is constant and no distinction is made between static and sliding friction.

After the introduction of the stick and slip constraints, one needs an indicator to decide whether stick or slip actually take place. Therefore an indicator function

$$f = \left\| \mathbf{t}_T \right\| - \mu \left| t_N \right| \tag{15}$$

is evaluated, which respect the Coulomb's model for frictional interface law. In the equation (15) the first term is $\left\| \mathbf{t}_T \right\| = \sqrt{t_{T\alpha} \overline{a}^{\alpha\beta} t_{T\beta}}$. Then the following contact states can be distinguished:

$$f = \begin{cases} \left\| \mathbf{t}_T \right\| - \mu \left| t_N \right| \leq 0 & \rightarrow \text{Stick} \\ \left\| \mathbf{t}_T \right\| - \mu \left| t_N \right| > 0 & \rightarrow \text{Slip} \end{cases} \tag{16}$$

Using the penalty method for normal stress, constitutive equation can be formulated as

$$t_N = \varepsilon_N g_N \tag{17}$$

where ε_N is the normal penalty parameter.

The tangential part is different for the stick and for the slip case. For stick a simple linear constitutive model can be used to describe the tangential stress

$$t_{T\alpha}^{stick} = \varepsilon_T g_{T\alpha} \tag{18}$$

where ε_T is the tangential penalty parameter. For slip the tangential stress given by the constitutive law for frictional sliding (14). A backward Euler integration scheme and return mapping strategy are employed to integrate the friction equations (15). Is a state of stick is assumed, the trial values of the tangential contact pressure vector $t_{T\alpha}$, and the indicator function f at load step *n+1* can be expressed in terms of their values at load step *n* as follows

$$t_{T\alpha\, n+1}^{trial} = t_{T\alpha\, n} + \varepsilon_T \Delta g_{T\alpha\, n+1} = t_{T\alpha\, n} + \varepsilon_T \overline{a}_{\alpha\beta} \Delta \xi_{n+1}^{\beta} \tag{19}$$

$$f_{Tn+1}^{trial} = \left\| \mathbf{t}_{Tn+1}^{trial} \right\| - \mu \left| t_{Nn+1} \right| \tag{20}$$

The return mapping is completed by

$$t_{T\alpha\, n+1} = \begin{cases} t_{T\alpha\, n+1}^{trial} & \text{if } f \leq 0 \\ \mu \left| t_{Nn+1} \right| n_{T\alpha\, n+1}^{trial} & \text{if } f > 0 \end{cases} \tag{21}$$

with

$$n_{T\alpha\, n+1}^{trial} = \frac{t_{T\alpha\, n+1}^{trial}}{\left\| \mathbf{t}_{Tn+1}^{trial} \right\|} \tag{22}$$

For the both cases, the penalty method can be illustrated as a group of linear elastic springs that force the body back to the contact surface when overlapping or sliding occurs.

3. Finite element formulation

3.1 Standard FE formulation without contact

Standard finite element approximations gives in case of dynamics a nonlinear system of differential equations:

$$\mathbf{M}\ddot{\mathbf{U}}+\mathbf{K}\mathbf{U}=\mathbf{F}(t) \tag{23}$$

where are: mass matrix

$$\mathbf{M}=\int_V \rho\mathbf{H}^T\mathbf{H}dV \tag{24}$$

stiffness matrix

$$\mathbf{K}=\int_V \mathbf{B}^T\mathbf{C}\mathbf{B}dV \tag{25}$$

and vector $\mathbf{F}(t)$ correspond to an external force. Here are: C - constitutive matrix, H - contains the shape functions and B strain-displacement matrix.

3.2 Finite element formulation of frictional contact

The virtual work of boundary nodes which are in contact is now formulated for one slave node k:

$$\delta A_c = F_N\delta g_N + \mathbf{F}_T\delta \mathbf{g}_T = t_N A_k \delta g_N + \mathbf{t}_T A_k \delta \mathbf{g}_T = t_N A_k \delta g_N + t_{T\alpha} A_k \delta \overline{\xi}^{\alpha} \tag{26}$$

Here are: $F_N = t_N A_k$ the normal force; $F_{T\alpha} = t_{T\alpha} A_k$ the tangential force [8]; A_k the area of the contact element; δg_N the virtual normal displacement (gap) and $\delta g_{T\alpha}$ the virtual tangential displacements.

Using equations (4) and (9) the variations of the gap and the tangential displacements in (26) are given by

$$\delta g_N = (\delta \mathbf{u}^k - \delta \overline{\mathbf{u}})\cdot \overline{\mathbf{n}} \tag{27}$$

$$\delta \overline{\xi}^{\alpha} = \overline{a}^{\alpha\beta}\left[\delta \mathbf{u}^k - \delta \overline{\mathbf{u}}\right]\cdot \overline{\mathbf{a}}_{\beta} \tag{28}$$

For the finite element implementations we need the matrix form of equations (26)-(28). For this purpose we define a displacement vector for the five-node contact elements (k, 1, 2, 3, 4)

$$\delta \mathbf{u}_c^T = \left\{\delta \mathbf{u}^k \quad \delta \mathbf{u}_1 \quad \delta \mathbf{u}_2 \quad \delta \mathbf{u}_3 \quad \delta \mathbf{u}_4\right\} \tag{29}$$

and the vectors

$$\mathbf{N}=\begin{Bmatrix} \overline{\mathbf{n}} \\ -H_1\overline{\mathbf{n}} \\ -H_2\overline{\mathbf{n}} \\ -H_3\overline{\mathbf{n}} \\ -H_4\overline{\mathbf{n}} \end{Bmatrix} \qquad \mathbf{T}_{\beta}=\begin{Bmatrix} \overline{\mathbf{a}}_{\beta} \\ -H_1\overline{\mathbf{a}}_{\beta} \\ -H_2\overline{\mathbf{a}}_{\beta} \\ -H_3\overline{\mathbf{a}}_{\beta} \\ -H_4\overline{\mathbf{a}}_{\beta} \end{Bmatrix} \qquad \mathbf{D}^{\alpha} = \overline{a}^{\alpha\beta}\mathbf{T}_{\beta} \tag{30}$$

With these definitions we obtain the matrix formulation of equations (27) and (28).

$$\delta g_N = \delta \mathbf{u}_c^T \cdot \mathbf{N}, \quad \delta \overline{\xi}^{\alpha} = \delta \mathbf{u}_c^T \cdot \mathbf{D}^{\alpha} \tag{31}$$

Thus the virtual work which has to be added to the standard finite element equation (23), can be expressed with (26) for one slave node k which is in contact, by

$$\mathbf{F}_c = \left[F_N\mathbf{N} + F_{T\alpha}\mathbf{D}^{\alpha}\right] \tag{32}$$

The contact forces F_N and $F_{T\alpha}$ in (32) can be obtain by multiplying the constitutive interfaces laws (17), (18) and (21) by the area of the contact element A_k. Finally, we obtain the global nonlinear finite element equation extended by contact forces (32)

$$\mathbf{M}\ddot{\mathbf{U}}+\mathbf{K}\mathbf{U}=\mathbf{F}(t)-\mathbf{F}_c \tag{33}$$

3.3 Algorithm for frictional contact

As can be seen from equation (33) we have to solve a nonlinear equilibrium equation with inequality constraints (6) as a result of contact. In order to apply Newton's method for the solution system of equilibrium equation (33), a linearization of the contact contributions is necessary. The linearization of the first term of (26), for the infinitesimal theory, gives

$$\Delta(t_N \delta g_N) = \frac{\partial t_N}{\partial g_N} \Delta g_N \delta g_N = \delta \mathbf{u}_c^T \mathbf{K}_N \Delta \mathbf{u}_c \tag{34}$$

It is assumed that the contact area A_k is not changing significantly. so the area A_k is contained within the penalty parameters. Using equations (17) and (31) we obtain

$$\frac{\partial t_N}{\partial g_N} = \varepsilon_N, \qquad \delta g_N \Delta g_N = \delta \mathbf{u}_c^T \mathbf{N} \mathbf{N}^T \Delta \mathbf{u}_c \tag{35}$$

Inserting (35) in (34) gives tangent stiffness matrix for the normal contact

$$\mathbf{K}_N = \varepsilon_N \mathbf{N} \mathbf{N}^T \tag{36}$$

Analogous to (34) we obtain the linearization for of the second term of (26)

$$\Delta(t_{T\alpha} \delta \bar{\xi}^\alpha) = \Delta t_{T\alpha}^{stick/slip} \delta \bar{\xi}^\alpha = \delta \mathbf{u}_c^T \mathbf{K}_T^{stick/slip} \Delta \mathbf{u}_c \tag{37}$$

For stick condition, using (9), we obtain

$$\Delta t_{T\alpha}^{stick} = \frac{\partial t_{T\alpha}^{stick}}{\partial g_{T\alpha}} \Delta g_{T\alpha} = \frac{\partial t_{T\alpha}^{stick}}{\partial g_{T\alpha}} \bar{a}_{\alpha\beta} \Delta \bar{\xi}^\beta \tag{38}$$

Using equations (18) and (31) we obtain

$$\frac{\partial t_{T\alpha}^{stick}}{\partial g_{T\alpha}} = \varepsilon_T, \qquad \delta \bar{\xi}^\alpha \Delta \bar{\xi}^\alpha = \delta \mathbf{u}_c^T \mathbf{D}^\alpha \mathbf{D}^{\beta T} \Delta \mathbf{u}_c \tag{39}$$

Inserting (38) and (39) in (37) gives the symmetric tangent stiffness matrix for stick condition

$$\mathbf{K}_T^{stick} = \varepsilon_T \bar{a}_{\alpha\beta} \mathbf{D}^\alpha \mathbf{D}^{\beta T} \tag{40}$$

For slip condition using (21) and (17) the linearization of $\Delta t_{T\alpha}^{slip}$ gives

$$\Delta t_{T\alpha}^{slip} = \Delta\left(\mu \left|\varepsilon_N g_{Nn+1}\right| n_{T\alpha\, n+1}^{trial}\right) = \mu \varepsilon_N \Delta g_{Nn+1} n_{T\alpha\, n+1}^{trial} + \mu \varepsilon_N g_{Nn+1} \Delta\left(n_{T\alpha\, n+1}^{trial}\right) \tag{41}$$

Using (22) and (19) the linearization of $n_{T\alpha\, n+1}^{trial}$ gives (for details see [3])

$$\Delta\left(n_{T\alpha\, n+1}^{trial}\right) = \Delta\left(\frac{t_{T\alpha\, n+1}^{trial}}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|}\right) = \frac{1}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|}\left[\delta_\alpha^\beta - n_{T\alpha\, n+1}^{trial} n_{T\, n+1}^{trial\, \beta}\right] \Delta t_{T\beta\, n+1}^{trial} \tag{42}$$

Inserting (42) and (41) in (37) and using (39) we get the tangent stiffness matrix for slip condition

$$\mathbf{K}_T^{slip} = \mu \varepsilon_N n_{T\alpha\, n+1}^{trial} \mathbf{D}^\alpha \mathbf{N}^T + \frac{\mu \varepsilon_N g_{Nn+1}}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|} \varepsilon_T \bar{a}_{\beta\gamma} \left[\delta_\alpha^\beta - n_{T\alpha\, n+1}^{trial} n_{T\, n+1}^{trial\, \beta}\right] \mathbf{D}^\alpha \mathbf{D}^{\gamma T} \tag{43}$$

The second term, the tangent matrix is non-symmetric. This is because the Coulomb's of friction can be viewed as a non-associative constitutive equation.

Frictional contact algorithm using penalty method is shown in Table 1.

LOOP over all contact segment k
 (check for contact (6)) IF $g_N \le 0$ THEN
  (the first iteration) IF $i=1$ THEN
   set all active nodes to state stick,
   $\mathbf{t}_{Tn+1}$ (18), compute matrix $\mathbf{K}_T^{stick}$
  ELSE
   Compute trial state: $t_{T\alpha\, n+1}^{trial}$ (19) and f_{Tn+1}^{trial} (20)
   IF $f_{Tn+1}^{trial} \le 0$ THEN
    $t_{T\alpha\, n+1} = t_{T\alpha\, n+1}^{trial}$, compute matrix $\mathbf{K}_T^{stick}$ (40)
    GO TO (a)
   ELSE
    $t_{T\alpha\, n+1} = \mu \left|t_{Nn+1}\right| n_{T\alpha\, n+1}^{trial}$, compute matrix $\mathbf{K}_T^{slip}$ (43)
   ENDIF
  ENDIF
 ENDIF
(a) END LOOP

Table 1. Frictional contact algorithm using the penalty method

4. Example

Example 1. Compression of cylinder between two parallel plates is considered. Initial dimension of cylinder are: radius $r = 6.35mm$, height $h = 2r$. Elasto-plastic material model with following yield curve is used

$$\sigma_y = \sigma_{y0} + \left(\sigma_{y\infty} - \sigma_{y0}\right)\left(1 - e^{-\delta\bar{e}_p}\right) + H\delta\bar{e}_p \tag{44}$$

Material constants are: $E = 210.40$ GPa, $\nu = 0.3118$, $K = 164.206$ GPa, $G = 80.1938$ GPa, $\sigma_{y0} = 0.45$ GPa, $\sigma_{y\infty} = 0.75$ GPa, $\delta = 16.96$ GPa, $H = 0.12924$ GPa. Due to symmetry, one-eight of the cylinder is modeled, with symmetry conditions for nodes lying in coordinate planes. The is problem considered without friction, using contact element based on the Lagrange (see [1]) and penalty methods.

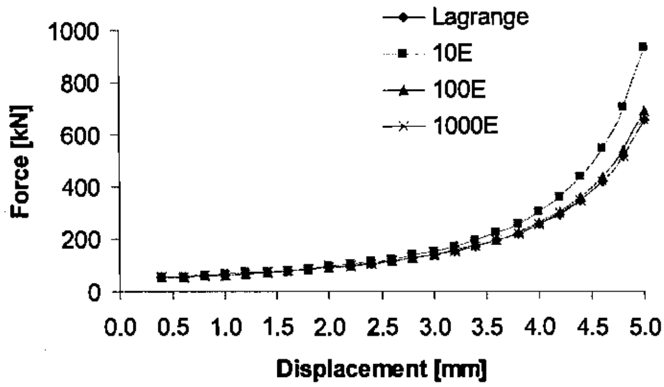

Fig. 2. Force – displacement relationship

Deformation of the cylinder is increased by prescribed displacement at the plate. Solution is obtained by 25 steps of displacement increments equal to 0.2 mm; and by full-Newton iteration method with line search. The three different values for normal penalty parameter are considered: a) $\varepsilon_N = 10 \cdot E$; b) $\varepsilon_N = 100 \cdot E$ and c) $\varepsilon_N = 1000 \cdot E$. The force - displacement diagram is shown in Fig. 2. In this example, a penalty number which is chosen have to be at least 100 times greater then E, for good approximation of the normal force. It is obvious, that the value of the penalty parameter impact on accuracy of the results in contact problems. Initial and deformed configuration at final step is shown in Fig. 3.

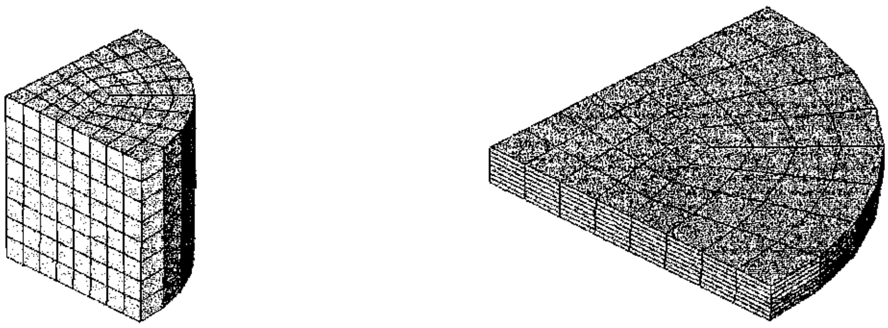

Fig. 3. Initial and final deformed configuration

5. Conclusions

A model for three-dimensional contact problem with friction based on the penalty method was proposed. Due to the intrinsic similarity between friction and classical elaso-plasticity the constitutive model for friction can be constructed following the same formalism. The example is showed indicate possibility applying developed method in analysis problem of finite deformation.

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